國立臺中教育大學 114 學年度日間部學士班轉學生招生考試

微積分試題

【①本考科得使用鉛筆；②並請一律於答案卷上作答】

適用學系：數學教育學系 二、三年級

一、填充題（每題 5%，共 55%）

1. Evaluate $\int \frac{1}{1+e^x}\,dx =$ ________.

2. Evaluate $\frac{d}{dx}\left[ln\left|\frac{cos\,x}{cos\,x-2}\right|\right] =$ __________.

3. Evaluate $\lim_{x\to\infty}\left(2xsin\frac{2}{x}\right) =$ __________.

4. Evaluate $\int_1^2 \frac{e^{5/x}}{x^2}\,dx =$ _________.

5. Evaluate $\int \frac{1}{x(ln(x^2))^3}\,dx =$ _________.

6. Evaluate $\int \frac{x^2-6x+2}{x^3+2x^2+x}\,dx =$ _________.

7. Evaluate $\lim_{x\to 3^-}\frac{x-\sqrt{3x}}{|x^2-9|} =$ ________.

8. Evaluate $\lim_{n\to\infty}\frac{\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\cdots+\frac{1}{\sqrt{n}}}{\sqrt{n}} =$ ________.

9. The equation of the tangent line to the curve $y^2(2-x)=x^3$ at the point (1,1) is ________.

10. The area of the surface generated by revolving the curve $y=x^3, 0\le x\le 2$ about the x-axis is ________.

11. Consider the curve defined implicitly by the equation $x^2e^{xy}+y=4$,

    The slope of the tangent line to the curve at the point (1,0) is __________.

（背面尚有試題）

二、計算證明題（**需書寫詳細計算證明過程**，共 45%）

1. Find the arc length of the curve defined by $x = \frac{1}{3}(y^2+2)^{3/2}$ on the interval $0 \le y \le 4$.(10%)

2. Find the general solution to the first-order differential equation $\frac{dy}{dx} = \frac{x-y}{x+y}$. (10%)

3. Please state carefully the ε, δ definition of a limit $\lim_{x\to a} f(x) = L$, and then prove $\lim_{x\to 2}(x^2-4x+5)=1$ using the ε, δ definition of a limit. (10%)

4. Let $f(x)=\frac{(x+1)^2}{1+x^2}$.(15%)
   (i) Find all asymptotes of the curve $y=f(x)$.
   (ii) Find the intervals on which f is increasing or decreasing.
   (iii) Find the intervals on which f is concave upward or concave downward.
   (iv) Find the local maximum and minimum values, and inflection points of f .
   (v) Use the information from parts (i)–(iv) to sketch the graph of $y=f(x)$.